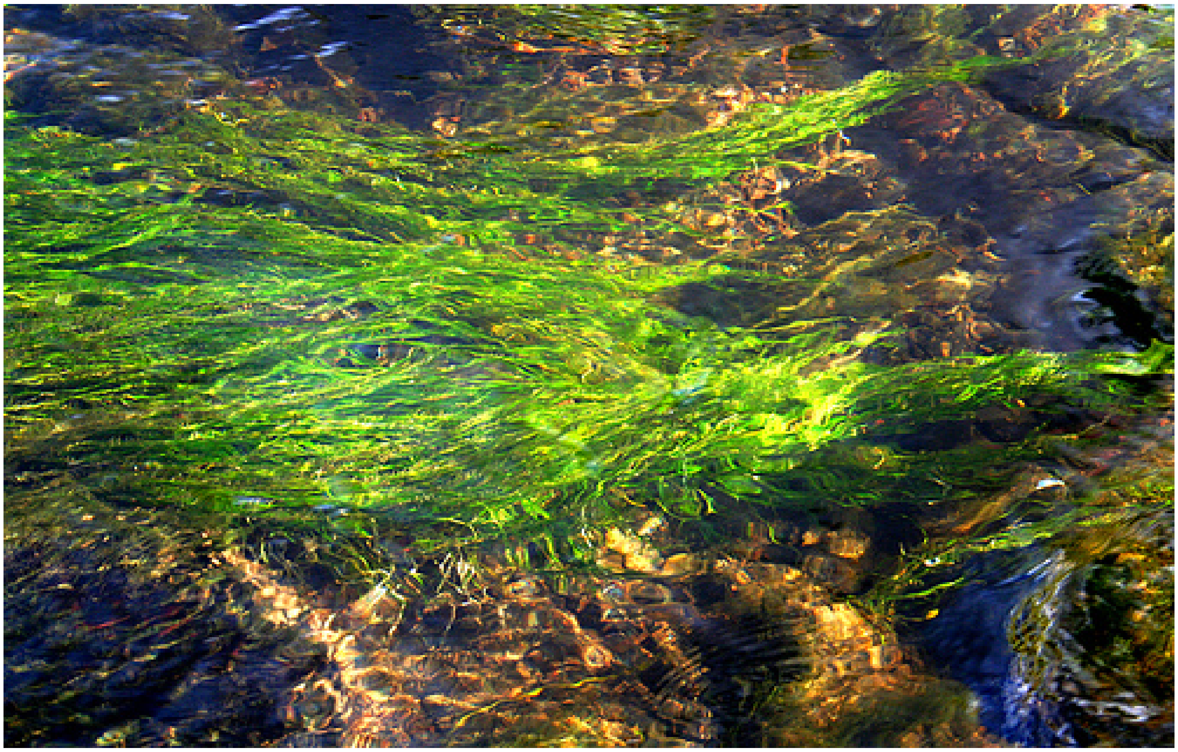
Documentation for

# PlantMap2K:
# A Bottom-Algae Cross-section Model



by

**Steven C. Chapra, Ph.D.**
E2K, LLC
Weston, MA 02493

prepared for

**Montana Department of Environmental Quality**
Watershed Modeling Program

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Montana Department of
Environmental Quality

E2K

## INTRODUCTION

A variety of water-quality modeling frameworks are available to assess the impact of excess nutrient inputs on river and stream eutrophication (e.g., Brown and Barnwell 1987, Shanahan et al. 2001, Chapra et al. 2009). Because these models are spatially one-dimensional, they generate depth-averaged and width-averaged predictions at locations along the stream's longitudinal dimension. Such predictions are appropriate for water-column variables which in many cases do not exhibit significant vertical and lateral gradients. For example, constituents such as dissolved oxygen and phytoplankton biomass are usually well-represented by cross-section averages.

In contrast, due to their dependence on light intensity, attached plants typically exhibit lateral heterogeneity with higher densities at shallower depths. The water-quality significance of this feature is reinforced because human use and perception is often inclined towards the shallows. As a consequence, there is a need to relate longitudinal model output to lateral plant densities.

One approach for accomplishing this objective would be to develop a three-dimensional water-quality model. Although this is certainly theoretically feasible, software is currently unavailable to make such computations. Further, the data requirements of such an approach would be extremely costly and hence impractical for most management applications.

The present report describes a simpler approach for making such calculations. It involves using the output of a one-dimensional longitudinal water-quality model, QUAL2K (Chapra et al. 2009) to drive a one-dimensional lateral bottom-plant model. The latter model, called PlantMap2K, generates bottom-algae biomass levels at locations across the stream width. Along with these levels, the software also computes various water-quality metrics including the percent of stream width with biomass levels exceeding numeric criteria.

## MODEL DESCRIPTION

### Cross-section Representation

As depicted in Figure 1, the cross-section is characterized by a series of depth measurements at discrete locations across the stream width. The model linearly interpolates between these soundings to generate a finer, uniformly-spaced depth profile.

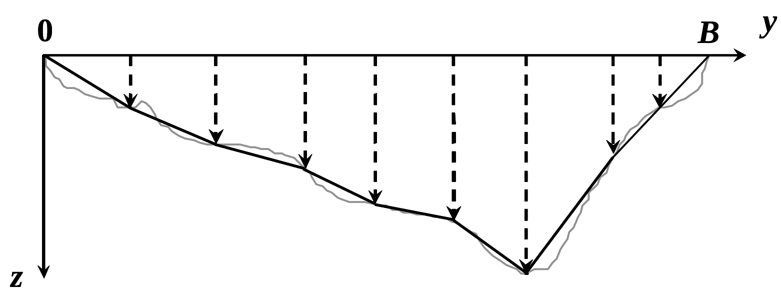



**Figure 1 Stream cross-section.**

Bottom plant densities are then computed at each of these locations using the bottom-algae algorithms from the QUAL2K submodel. In so doing, the submodel assumes that the water is sufficiently well-mixed vertically and horizontally. Hence, the Q2K output for the water-column at the particular cross-section location serves as the boundary condition for the bottom algae regardless of their lateral location. However, because solar radiation is attenuated with depth, different bottom plant biomass concentrations are generated at points across the stream width.

It should be noted that although this assumption of a homogeneous water column often holds, it would be violated immediately downstream from major point sources such as sewage effluents or tributary inflows. Appendix 1 provides guidance on assessing its validity.

## Algorithm

Rather than running the Q2K bottom-algae submodel for each depth across the stream width, the algorithm first develops a table of mean biomass levels as a function of depth. This is done by running the Q2K bottom-algae submodel for a number of depth increments ranging from 0 to the cross-section's maximum depth. Each run is executed for the number of days specified by the user and on the final day, the mean biomass is computed. Linear interpolation is then used to generate the mean biomass level for each depth across the stream cross-section.

## Model Equations

QUAL2K's bottom-algae submodel consists of 3 state variables:

- Bottom-algae biomass, $a_b$, mgA/m$^2$
- Bottom-algae internal phosphorus, $IP_b$, mgP/m2
- Bottom-algae internal nitrogen, $IN_b$, mgN/m$^2$

In addition, the cell quotas for phosphorus, $q_{Pb}$ (mgP/mgA), and nitrogen, $q_{Nb}$ (mgN/mgA), are computed from ratios of the state variables as $q_{Pb} = IP_b/a_b$ and $q_{Nb} = IN_b/a_b$. The model equations used to simulate the dynamics of the state variables are identical to those described in the Q2K documentation (Chapra et al. 2010). For the reader's convenience, these and supporting equations are outlined in App. .

# USERS MANUAL

## Overview

The computer code used to implement the calculations for PlantMap2K (PM2K) is written in Visual Basic for Applications (VBA). Excel serves as the user interface.

Color is used to signify whether information is to be input by the user or output by the program:

- Pale Blue designates variable and parameter values that are to be entered by the user.
- Pale Yellow designates data that the user enters. This data are then displayed on graphs generated by PM2K.
- Pale Green designates output values generated by PM2K.
- Dark solid colors are used for labels and should not be changed.

All worksheets include two buttons (Figure 1):

- RUN. This button causes PM2K to execute and to create a data file that holds the input values. The data file can then be accessed later using the Open Old File button.

- OPEN FILE. When this button is clicked, the file browser will automatically open to allow you to access a data file created on a previous model run. All PlantMap2K data files have the extension, *.px2.

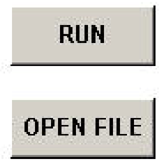
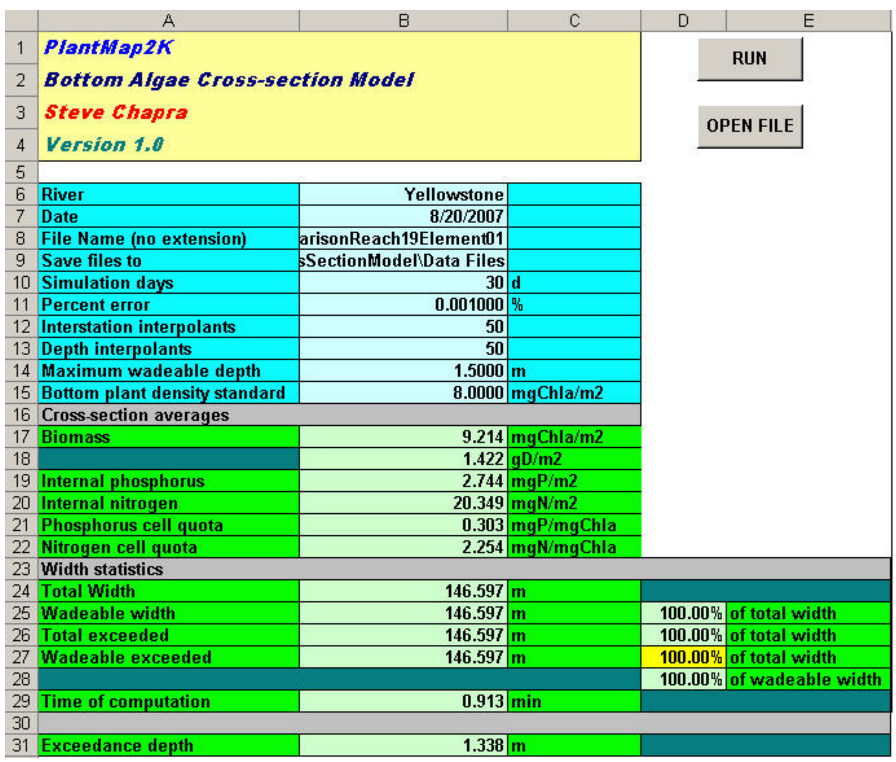



**Figure 2  The buttons to operate PM2K.**

## Worksheets

### PlantMap2K Worksheet (Input/Output)

The *PlantMap2K Worksheet* (Figure 3) is used to enter general information regarding a particular model application.



| | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | PlantMap2K | | | RUN | |
| 2 | Bottom Algae Cross-section Model | | | | |
| 3 | Steve Chapra | | | OPEN FILE | |
| 4 | Version 1.0 | | | | |
| 5 | | | | | |
| 6 | River | Yellowstone | | | |
| 7 | Date | 8/20/2007 | | | |
| 8 | File Name (no extension) | arisonReach19Element01 | | | |
| 9 | Save files to | sSectionModel\Data Files | | | |
| 10 | Simulation days | 30 | d | | |
| 11 | Percent error | 0.001000 | % | | |
| 12 | Interstation interpolants | 50 | | | |
| 13 | Depth interpolants | 50 | | | |
| 14 | Maximum wadeable depth | 1.5000 | m | | |
| 15 | Bottom plant density standard | 8.0000 | mgChla/m2 | | |
| 16 | Cross-section averages | | | | |
| 17 | Biomass | 9.214 | mgChla/m2 | | |
| 18 | | 1.422 | gD/m2 | | |
| 19 | Internal phosphorus | 2.744 | mgP/m2 | | |
| 20 | Internal nitrogen | 20.349 | mgN/m2 | | |
| 21 | Phosphorus cell quota | 0.303 | mgP/mgChla | | |
| 22 | Nitrogen cell quota | 2.254 | mgN/mgChla | | |
| 23 | Width statistics | | | | |
| 24 | Total Width | 146.597 | m | | |
| 25 | Wadeable width | 146.597 | m | 100.00% | of total width |
| 26 | Total exceeded | 146.597 | m | 100.00% | of total width |
| 27 | Wadeable exceeded | 146.597 | m | 100.00% | of total width |
| 28 | | | | 100.00% | of wadeable width |
| 29 | Time of computation | 0.913 | min | | |
| 30 | | | | | |
| 31 | Exceedance depth | 1.338 | m | | |

**Figure 3  The PlantMap2K Worksheet.**

<u>**Inputs**</u>

**River.** Name of the river or stream being modeled. After the program is run, this name along with the date, is displayed on all sheets and charts.
**Date.** Date on which bottom-algae biomass is being simulated.

**File name.** This is the name of the data file generated when PM2K is run, and which contains all information entered by the user. The software appends the extension *.px2 to this file name in order that the software can recognize it for future input (using the Open File button).
**Directory where file saved.** This specifies the complete path to the directory where you want the data file to be saved.
**Simulation days.** The number of days you want the model to run. Because the model starts with all variables at zero, it is important that the simulation days be set sufficiently high that a steady state condition is attained.
**Percent error.** This value sets an accuracy for computing the steady-state solution. If you leave it blank, a default value of 0.0001% is automatically set. If this accuracy is achieved, the model terminates before the simulation days are reached. If this accuracy is not achieved, the model terminates when the simulation days are reached.
**Interstation interpolants.** This value sets the number of interpolated values used to generate intermediate values between sample locations. If this cell is left blank, a default value of 50 is automatically set.
**Depth interpolants.** This value sets the number of interpolated values used to generate intermediate values between zero depth and a value 10% deeper than the maximum cross-section depth. If this cell is left blank, a default value of 50 is automatically set.
**Maximum wadeable depth.** This is the depth beyond which it is assumed that individuals cannot wade safely.
**Bottom plant density standard.** This is the level of bottom-algae biomass that is considered a violation.

## <u>Outputs</u>

**Cross-section averages.** Average daily values for the entire cross-section are displayed for total bottom-algae biomass as well as for internal nutrient content.
**Total width.** The total channel width.
**Wadeable width.** The extent of the channel width that is wadeable is reported as meters and as percent of total width.
**Total exceeded.** The extent of the channel width for which the bottom plant density standard is exceeded is reported as both meters and as percent of total width.
**Wadeable exceeded.** The extent of the wadeable width for which the bottom plant density standard is exceeded is reported as meters, percent of total width and percent of wadeable width.
**Time of computation.** The model run time.
**Exceedence depth.** The depth beyond which exceedences do not occur.

## <u>Plots</u>

Aside from entering and displaying values, the PlantMap2K sheet also displays two plots. As in Figure 4, these are (a) the depth profile showing the wadeable and violation depths, and (b) the profile of the average daily bottom-algae biomass showing the violation standard.

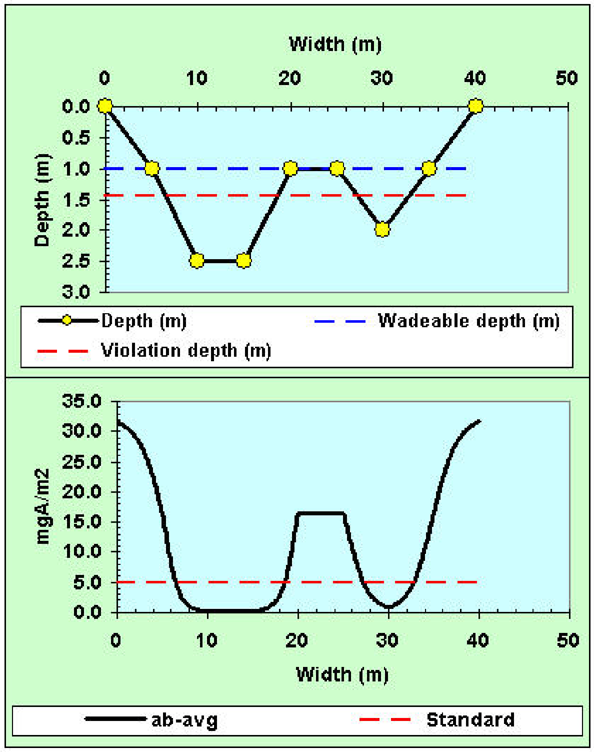
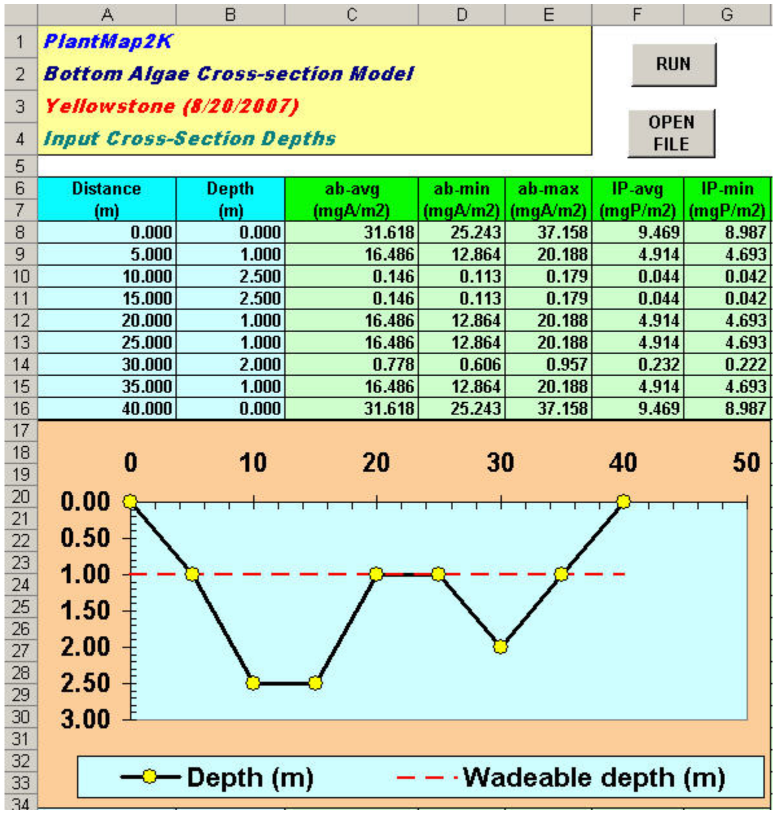



**Figure 4 Plots displayed on the PlantMap2K Worksheet.**

## Input Cross-section Worksheet (Input/Output)

The *Input Cross-section Worksheet* (Figure 5) is used to enter the cross-section depth profile. It also displays the depth profile as well as the numeric values of the bottom-algae biomass at the locations where the depths are specified.



| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | *PlantMap2K* | | | | | RUN | |
| 2 | *Bottom Algae Cross-section Model* | | | | | | |
| 3 | *Yellowstone (8/20/2007)* | | | | | OPEN FILE | |
| 4 | *Input Cross-Section Depths* | | | | | | |
| 5 | | | | | | | |
| 6–7 | Distance (m) | Depth (m) | ab-avg (mgA/m2) | ab-min (mgA/m2) | ab-max (mgA/m2) | IP-avg (mgP/m2) | IP-min (mgP/m2) |
| 8 | 0.000 | 0.000 | 31.618 | 25.243 | 37.158 | 9.469 | 8.987 |
| 9 | 5.000 | 1.000 | 16.486 | 12.864 | 20.188 | 4.914 | 4.693 |
| 10 | 10.000 | 2.500 | 0.146 | 0.113 | 0.179 | 0.044 | 0.042 |
| 11 | 15.000 | 2.500 | 0.146 | 0.113 | 0.179 | 0.044 | 0.042 |
| 12 | 20.000 | 1.000 | 16.486 | 12.864 | 20.188 | 4.914 | 4.693 |
| 13 | 25.000 | 1.000 | 16.486 | 12.864 | 20.188 | 4.914 | 4.693 |
| 14 | 30.000 | 2.000 | 0.778 | 0.606 | 0.957 | 0.232 | 0.222 |
| 15 | 35.000 | 1.000 | 16.486 | 12.864 | 20.188 | 4.914 | 4.693 |
| 16 | 40.000 | 0.000 | 31.618 | 25.243 | 37.158 | 9.469 | 8.987 |



**Figure 5  Input cross-section worksheet.**

**<u>Input</u>**

The cross-section depth profile is entered as the distance from the bank (column A) and the corresponding depth (column B).

**<u>Output</u>**

The simulated bottom-algae biomass is displayed at the locations where the depths are specified (columns C through P). Average, minimum and maximum values are displayed for biomass, internal nutrients and cell quotes.

**<u>Plot</u>**

A plot of depth versus distance from the bank is displayed with the wadeable depth indicated.

## Light Parameters Worksheet (Input)

The *Light Parameters Worksheet* (Figure 6) is used to enter the parameters governing light extinction that were employed in the QUAL2K model. The values entered on this sheet should be identical to those on the corresponding cells of the QUAL2K *Light and Heat Worksheet*.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | PlantMap2K | | | |
| 2 | Bottom Algae Cross-section Model | | | |
| 3 | Yellowstone (8/20/2007) | | | |
| 4 | Light Parameters | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | Parameter | Value | Unit | |
| 8 | Photosynthetically Available Radiation | 0.47 | | |
| 9 | Background light extinction | 0.3 | /m | $k_{eb}$ |
| 10 | Linear chlorophyll light extinction | 0.0088 | 1/m-(ugA/L) | $\alpha_p$ |
| 11 | Nonlinear chlorophyll light extinction | 0.054 | 1/m-(ugA/L)2/3 | $\alpha_{pn}$ |
| 12 | ISS light extinction | 0.052 | 1/m-(mgD/L) | $\alpha_i$ |
| 13 | Detritus light extinction | 0.174 | 1/m-(mgD/L) | $\alpha_o$ |

**Figure 6 Light Parameters Worksheet.**

## Rates Worksheet (Input)

The *Rates Worksheet* (Figure 7) is used to enter the parameters governing bottom-algae stoichiometry and kinetics that were employed in the QUAL2K model. The values entered on this sheet should be identical to those on the corresponding cells of the QUAL2K *Rates Worksheet*.

| | A | B | C | D |
|---|---|---|---|---|
| 1 | ***PlantMap2K*** | | | |
| 2 | ***Bottom Algae Cross-section Model*** | | | |
| 3 | ***Yellowstone (8/20/2007)*** | | | |
| 4 | ***Rates*** | | | |
| 5 | | | | |
| 6 | | | | |
| 7 | *Parameter* | *Value* | *Units* | *Symbol* |
| 8 | *Stoichiometry:* | | | |
| 9 | Carbon | 43 | gC | gC |
| 10 | Nitrogen | 3.7 | gN | gN |
| 11 | Phosphorus | 1 | gP | gP |
| 12 | Dry weight | 108 | gD | gD |
| 13 | Chlorophyll | 0.7 | gA | gA |
| 14 | *Bottom Algae:* | | | |
| 15 | Growth model | Zero-order | | |
| 16 | Max Growth rate | 130 | mgA/m²/d or /d | $C_{gb}$ |
| 17 | Temp correction | 1.07 | | $\theta_{gb}$ |
| 18 | First-order model carrying capacity | 1000 | mgA/m² | $a_{b,max}$ |
| 19 | Respiration rate | 0.4 | /d | $k_{rb}$ |
| 20 | Temp correction | 1.07 | | $\theta_{rb}$ |
| 21 | Excretion rate | 0.4 | /d | $k_{eb}$ |
| 22 | Temp correction | 1.05 | | $\theta_{db}$ |
| 23 | Death rate | 0.4 | /d | $k_{db}$ |
| 24 | Temp correction | 1.07 | | $\theta_{db}$ |
| 25 | External nitrogen half sat constant | 370 | ugN/L | $k_{sPb}$ |
| 26 | External phosphorus half sat constant | 60 | ugP/L | $k_{sNb}$ |
| 27 | Inorganic carbon half sat constant | 1.30E-05 | moles/L | $k_{sCb}$ |
| 28 | Light model | Smith | | |
| 29 | Light constant | 40 | langley/d | $K_{Lb}$ |
| 30 | Ammonia preference | 1 | ugN/L | $k_{hnxb}$ |
| 31 | Subsistence quota for nitrogen | 0.72 | mgN/mgA | $q_{0N}$ |
| 32 | Subsistence quota for phosphorus | 0.1 | mgP/mgA | $q_{0P}$ |
| 33 | Maximum uptake rate for nitrogen | 890 | mgN/m²/d | $\rho_{mN}$ |
| 34 | Maximum uptake rate for phosphorus | 40 | mgP/m²/d | $\rho_{mP}$ |
| 35 | Internal nitrogen half sat constant | 0.9 | mgN/mgA | $K_{qN}$ |
| 36 | Internal phosphorus half sat constant | 0.13 | mgP/mgA | $K_{qP}$ |

**Figure 7 Rates Worksheet.**

## Forcing Function Worksheet (Input/Output)

The *Forcing Function Worksheet* (Figure 8) is used to enter the diel water-column characteristics that were generated with the QUAL2K model for the location of the cross-section being analyzed. The values entered on this sheet should be identical to those on the corresponding cells of the QUAL2K *Diel Output Worksheet*.

| | A | B | C | D | E | F |
|---|---|---|---|---|---|---|
| 1 | ***PlantMap2K*** | | | | | |
| 2 | ***Bottom Algae Cross-section Model*** | | | | RUN | |
| 3 | ***Yellowstone (8/20/2007)*** | | | | | |
| 4 | ***Q2K Diel Output for Element*** | | | | OPEN FILE | |
| 5 | | | | | | |
| 6 | Yellowstone (8/20/2007) | | | | | |
| 7 | Reach | 19 | | | | |
| 8 | Element | 1 | | | | |
| 9 | Averages--> | 21.2068 | 21.2078 | 759.1463 | 27.8912 | 8.6718 |
| 10 | | | | | | |
| 11 | *t (hr)* | *Tempw(C)* | *Temps(C)* | *cond (umhos)* | *ISS (mg/L)* | *DO(mg/L)* |
| 12 | 0.00 | 21.44 | 21.71 | 759.14 | 27.89 | 8.55 |
| 13 | 0.09 | 21.43 | 21.70 | 759.14 | 27.89 | 8.53 |
| 14 | 0.19 | 21.41 | 21.69 | 759.14 | 27.89 | 8.52 |
| 15 | 0.28 | 21.40 | 21.68 | 759.14 | 27.89 | 8.50 |
| 16 | 0.38 | 21.38 | 21.67 | 759.14 | 27.89 | 8.49 |
| 17 | 0.47 | 21.37 | 21.65 | 759.14 | 27.89 | 8.47 |
| 18 | 0.56 | 21.36 | 21.64 | 759.14 | 27.89 | 8.46 |
| 19 | 0.66 | 21.34 | 21.63 | 759.14 | 27.89 | 8.44 |

**Figure 8  Q2K diel inputs.**

## Data Worksheet (Input/Output)

The *Data Worksheet* (Figure 9) is used to enter measurements of mean, minimum and maximum biomass made at various locations across the stream width. The distance (column A) should be measured out from the same bank used to specify the depth profile on the *Input Cross-section Worksheet* (Figure 5). The distances do not have to correspond to the locations used to specify the depth profile. However, they must be within the bounds entered on the *Input Cross-section Worksheet*. An error message will be displayed if distances that are outside the bounds are entered.

| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | ***PlantMap2K*** | | | | | | | Standard | |
| 2 | ***Bottom Algae Cross-section Model*** | | | | | RUN | | 0.000 | 5.000 |
| 3 | ***Yellowstone (8/20/2007)*** | | | | | | | 2.500 | 5.000 |
| 4 | ***Data*** | | | | | OPEN FILE | | SSR | 188.026 |
| 5 | | | | | | | | Std Error | 4.848 |
| 6 | | | | | | | | r2 | 0.760 |
| 7 | | | | | | | | | |
| 8 | | | | | | | | | |
| 9–10 | Distance (m) | ab-avg (mgA/m2) | ab-min (mgA/m2) | ab-max (mgA/m2) | ab-avg-model (mgA/m2) | ab-min-model (mgA/m2) | ab-max-model (mgA/m2) | min error (mgA/m2) | max error (mgA/m2) |
| 11 | 4.000 | 17.000 | 10.000 | 20.000 | 22.334 | 17.447 | 27.151 | 7.000 | 3.000 |
| 12 | 11.000 | 0.300 | 0.100 | 0.400 | 0.146 | 0.113 | 0.179 | 0.200 | 0.100 |
| 13 | 14.500 | 0.150 | 0.100 | 0.300 | 0.146 | 0.113 | 0.179 | 0.050 | 0.150 |
| 14 | 19.000 | 16.000 | 14.000 | 18.000 | 7.974 | 6.227 | 9.815 | 2.000 | 2.000 |
| 15 | 26.000 | 17.000 | 12.000 | 25.000 | 10.690 | 8.346 | 13.138 | 5.000 | 8.000 |
| 16 | 29.000 | 0.500 | 0.600 | 0.600 | 1.519 | 1.184 | 1.869 | -0.100 | 0.100 |
| 17 | 36.000 | 15.000 | 13.000 | 19.000 | 22.334 | 17.447 | 27.151 | 2.000 | 4.000 |
| 18 | 39.000 | 30.000 | 24.000 | 40.000 | 30.698 | 24.529 | 36.214 | 6.000 | 10.000 |
| 19 | | | | | | | | | |

**Figure 9  Data Worksheet.**

The values entered on the *Data Worksheet* are displayed along with the model output on the *Biomass Plot* (Figure 12). Note that the biomass values in columns B through D can be left blank. However,

## Depth Calibration Curves Worksheet (Output)

The *Depth Calibration Curves Worksheet* (Figure 10) is used to enter measurements of mean, minimum and maximum biomass made at various locations across the stream width. The distance (column A) should be measured out from the same bank used to specify the depth profile on the *Input Cross-section Worksheet* (Figure 5). The distances do not have to correspond to the locations used to specify the depth profile. However, they must be within the bounds entered on the *Input Cross-section Worksheet*. An error message will be displayed if distances that are outside the bounds are entered.

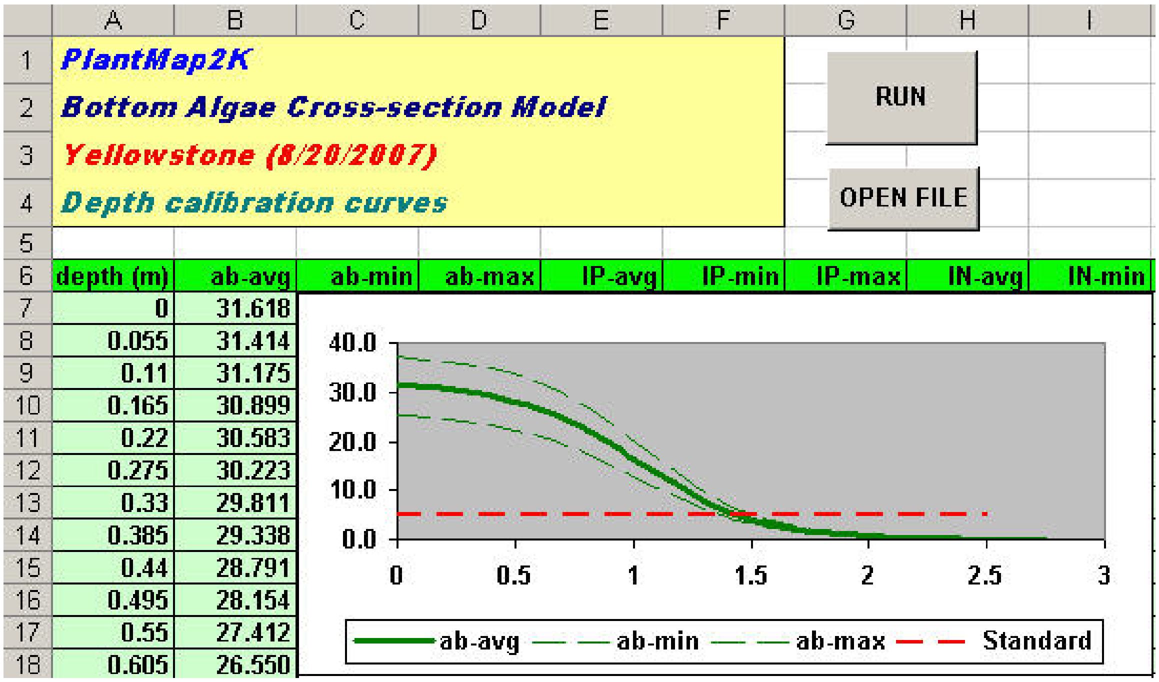



**Figure 10 Depth calibration curves.**

## Profile Plots Worksheet (Output)

The *Profile Plots Worksheet* (Figure 11) is used to enter measurements of mean, minimum and maximum biomass made at various locations across the stream width. The distance (column A) should be measured out from the same bank used to specify the depth profile on the *Input Cross-section Worksheet* (Figure 5). The distances do not have to correspond to the locations used to specify the depth profile. However, they must be within the bounds entered on the *Input Cross-section Worksheet*. An error message will be displayed if distances that are outside the bounds are entered.

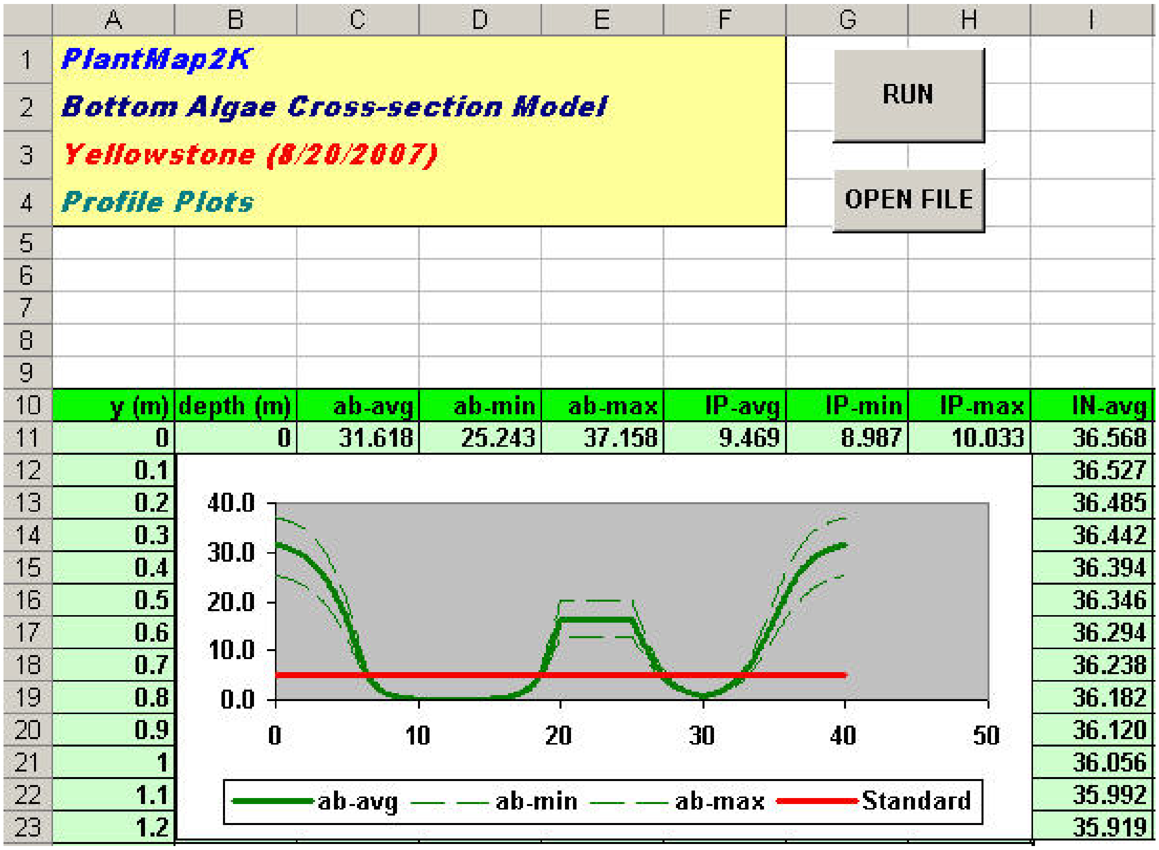



**Figure 11 Profile outputs and plots.**

Biomass Plot Chart (Output)

The *Biomass Plot Chart* (Figure 12) is used to enter measurements of mean, minimum and maximum biomass made at various locations across the stream width. The distance (column A) should be measured out from the same bank used to specify the depth profile on the *Input Cross-section Worksheet* (Figure 5). The distances do not have to correspond to the locations used to specify the depth profile. However, they must be within the bounds entered on the *Input Cross-section Worksheet*. An error message will be displayed if distances that are outside the bounds are entered.

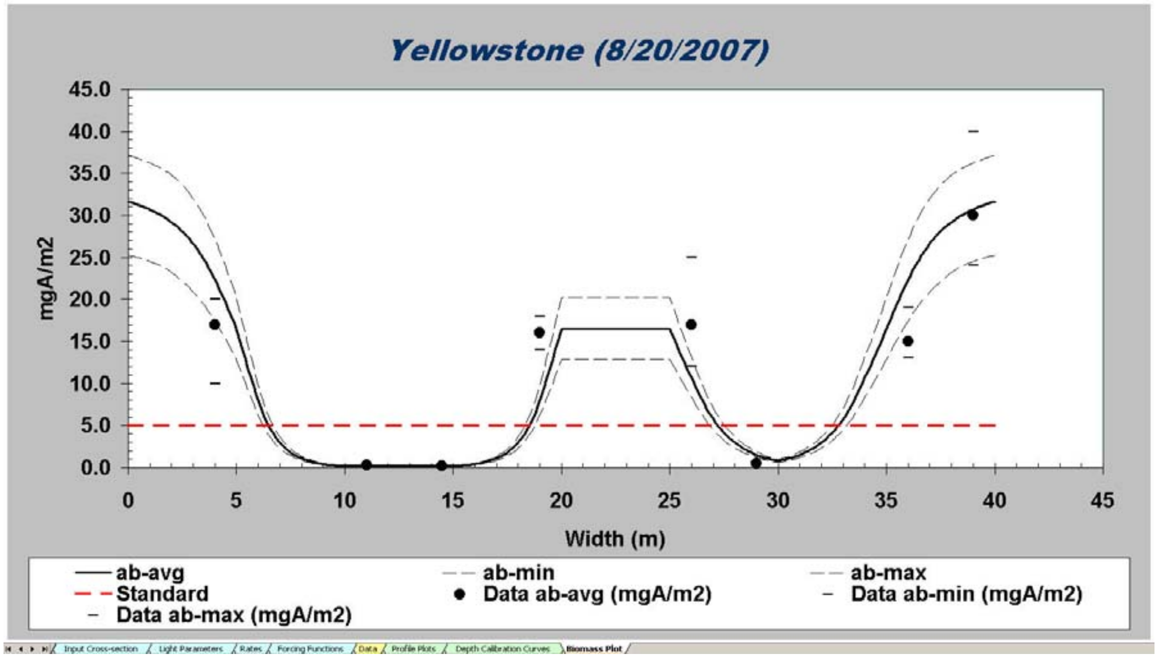



**Figure 12 Plot of biomass versus distance (m) across the cross-section. Both model results (lines) and data (points) are displayed**

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# APPENDIX A: Q2K Bottom-algae Submodel

## STATE VARIABLES

The Q2K bottom-algae submodel includes three state variables:

- Bottom-algae biomass, $a_b$, mgA/m$^2$
- Bottom-algae internal phosphorus, $IP_b$, mgP/m2
- Bottom-algae internal nitrogen, $IN_b$, mgN/m$^2$

In addition, the cell quotas for phosphorus, $q_{Pb}$ (mgP/mgA), and nitrogen, $q_{Nb}$ (mgN/mgA), can be computed from the state variables as

$$q_{Pb} = \frac{IP_b}{a_b} \tag{1}$$

$$q_{Nb} = \frac{IN_b}{a_b} \tag{2}$$

Mass balances for the state variables can be written as

$$\frac{da_b}{dt} = S_b \tag{3}$$

$$\frac{dIN_b}{dt} = S_{bN} \tag{4}$$

$$\frac{dIP_b}{dt} = S_{bP} \tag{5}$$

where $S_b$ = sources and sinks of bottom algae biomass due to reactions [mgA/m$^2$/d], $S_{bN}$ = sources and sinks of bottom algae nitrogen due to reactions [mgN/m$^2$/d], and $S_{bP}$ = sources and sinks of bottom algae phosphorus due to reactions [mgP/m$^2$/d].

## SOURCE/SINK TERMS

### Bottom algae ($a_b$)

Bottom algae increase due to photosynthesis. They are lost via respiration and death.

$$S_{ab} = \text{BotAlgPhoto} - \text{BotAlgResp} - \text{BotAlgDeath} \tag{6}$$

#### Photosynthesis

Bottom-algae photosynthesis is based on a temperature-corrected zero-order rate attenuated by internal nutrient levels and light limitation (McIntyre 1973, Rutherford et al. 1999),

$$\text{BotAlgPhoto} = C_{gb}(T)\phi_{Nb}\phi_{Lb} \quad (7)$$

where $C_{gb}(T)$ = the zero-order temperature-dependent maximum photosynthesis rate [mgA/(m$^2$ d)], $\phi_{Nb}$ = bottom-algae internal nutrient attenuation factor [dimensionless number between 0 and 1], and $\phi_{Lb}$ = the bottom-algae light attenuation coefficient [dimensionless number between 0 and 1].

**Temperature Effect.** An Arrhenius model is employed to quantify the effect of temperature on bottom-algae photosynthesis,

$$C_{gb}(T) = C_{gb}(20)\theta^{T-20} \quad (8)$$

**Nutrient Limitation.** The effect of nutrient limitation on bottom plant photosynthesis is modeled with a Droop (1974) formulation for nitrogen and phosphorus limitation and a Michaelis-Menten equation for inorganic carbon,

$$\phi_{Nb} = \min\left[1 - \frac{q_{0Nb}}{q_{Nb}}, 1 - \frac{q_{0Pb}}{q_{Pb}}, \frac{[\mathrm{H_2CO_3^*}] + [\mathrm{HCO_3^-}]}{k_{sCb} + [\mathrm{H_2CO_3^*}] + [\mathrm{HCO_3^-}]}\right] \quad (9)$$

where $q_{0Nb}$ and $q_{0Pb}$ = the bottom-algae minimum cell quotas of nitrogen [mgN mgA$^{-1}$] and phosphorus [mgP mgA$^{-1}$], respectively, and , $k_{sCb}$ = the bottom-algae inorganic carbon half-saturation constant [mole/L]. In addition, the code is designed so that the nutrient limitation terms cannot be negative.

**Light Limitation.** Light limitation at any time is determined by the amount of PAR reaching the bottom of the water column. This quantity is computed with the Beer-Lambert law evaluated at the river bottom,

$$PAR(z) = PAR(0)e^{-k_e H} \quad (10)$$

where $PAR(z)$ = photosynthetically available radiation (PAR) at depth $z$ below the water surface [ly/d][1], and $k_e$ = the light extinction coefficient [m$^{-1}$]. The PAR at the water surface is assumed to be a fixed fraction of the solar radiation (Szeicz 1984, Baker and Frouin 1987):

$$PAR(0) = 0.47\ I(0)$$

The extinction coefficient is related to model variables by

$$k_e = k_{eb} + \alpha_i m_i + \alpha_o m_o + \alpha_p a_p + \alpha_{pn} a_p^{2/3} \quad (11)$$

where $k_{eb}$ = the background coefficient accounting for extinction due to water and color [/m], $\alpha_i$, $\alpha_o$, $\alpha_p$, and $\alpha_{pn}$, are constants accounting for the impacts of inorganic suspended solids [L/mgD/m], particulate organic matter [L/mgD/m], and chlorophyll [L/μgA/m and (L/μgA)$^{2/3}$/m], respectively. Suggested values for these coefficients are listed in Table 1.

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[1] ly/d = langley per day. A langley is equal to a calorie per square centimeter. Note that a ly/d is related to the μE/m$^2$/d by the following approximation: 1 μE/m$^2$/s ≅ 0.45 Langley/day (LIC-OR, Lincoln, NE).

**Table 1 Suggested values for light extinction coefficients**

| Symbol | Value | Reference |
|---|---|---|
| $\alpha_i$ | 0.052 | Di Toro (1978) |
| $\alpha_o$ | 0.174 | Di Toro (1978) |
| $\alpha_p$ | 0.0088 | Riley (1956) |
| $\alpha_{pn}$ | 0.054 | Riley (1956) |

Three models are used to characterize the impact of light on photosynthesis (Figure 13):

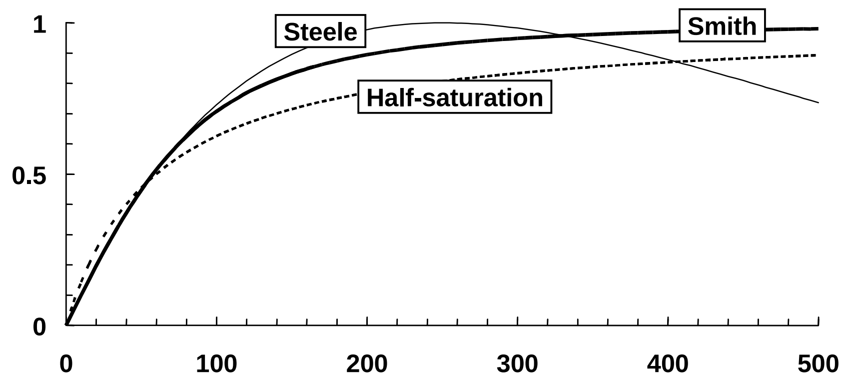


**Figure 13 The three models used for bottom-algae photosynthetic light dependence. The plot shows growth attenuation versus PAR intensity [ly/d].**

Half-Saturation (Michaelis-Menten) Light Model (Baly 1935):

$$\phi_{Lb} = \frac{PAR(0)e^{-k_e H}}{K_{Lb} + PAR(0)e^{-k_e H}} \tag{12}$$

where $K_{Lp}$ = the bottom-algae light half-saturation coefficient [ly/d].

Smith's Function (Smith 1936):

$$\phi_{Lb} = \frac{PAR(0)e^{-k_e H}}{\sqrt{K_{Lb}^2 + \left(PAR(0)e^{-k_e H}\right)^2}} \tag{13}$$

where $K_{Lb}$ = the Smith parameter for bottom algae [ly/d]; that is, the PAR at which growth is 70.7% of the maximum.

Steele's Equation (Steele 1962):

$$\phi_{Lb} = \frac{PAR(0)e^{-k_e H}}{K_{Lb}} e^{1+\frac{PAR(0)e^{-k_e H}}{K_{Lb}}} \tag{14}$$

where $K_{Lb}$ = the PAR at which bottom-algae growth is optimal [ly/d].

**Space Limitation.** If a first-order growth model is used, a term must be included to impose a space limitation on the bottom algae. A logistic model is used for this purpose as in

$$\phi_{Sb} = 1 - \frac{a_b}{a_{b,\max}}$$

where $a_{b,\max}$ = the carrying capacity [mgA/m$^2$].

## Losses

**Respiration.** Bottom-algae respiration is represented as a first-order rate that is attenuated at low oxygen concentration,

$$\text{BotAlgResp} = F_{oxb} k_{rb}(T)\, a_b \tag{15}$$

where $k_{rb}(T)$ = temperature-dependent bottom-algae respiration rate [/d] and $F_{oxb}$ = attenuation due to low oxygen [dimensionless]. Oxygen attenuation is modeled with the following relationships with the oxygen dependency represented by the parameter $K_{sob}$,

<u>Half-Saturation:</u>

$$F_{oxrp} = \frac{o}{K_{socf} + o} \tag{16}$$

where $K_{socf}$ = half-saturation constant for the effect of oxygen on fast CBOD oxidation [mg$O_2$/L].

<u>Exponential:</u>

$$F_{oxrp} = (1 - e^{-K_{socf} o}) \tag{17}$$

where $K_{socf}$ = exponential coefficient for the effect of oxygen on fast CBOD oxidation [L/mg$O_2$].

<u>Second-Order Half Saturation:</u>

$$F_{oxrp} = \frac{o^2}{K_{socf} + o^2} \tag{18}$$

where $K_{socf}$ = half-saturation constant for second-order effect of oxygen on fast CBOD oxidation [mg$O_2^2$/$L^2$].

**Death.** Bottom-algae death is represented as a first-order rate,

$$\text{BotAlgDeath} = k_{db}(T) a_b \tag{19}$$

where $k_{db}(T)$ = the temperature-dependent bottom-algae death rate [/d].

## Bottom-algae Internal Nitrogen (*IN_b*)

The change in intracellular nitrogen in bottom algal cells is calculated from

$$S_{bN} = \text{BotAlgUpN} - q_{Nb}\ \text{BotAlgDeath} - \text{BotAlgExN} \tag{20}$$

where BotAlgUpN = the maximum uptake rate of nitrogen by bottom algae (mgN/m$^2$/d), BotAlgDeath = bottom-algae death (mgA/m$^2$/d), and BotAlgExN = the bottom-algae excretion of nitrogen (mgN/m$^2$/d), which is computed as

$$\text{BotAlgExN} = k_{eb}(T)(q_{Nb} - q_{0Nb})a_b \tag{21}$$

where $k_{eb}(T)$ = the temperature-dependent bottom-algae excretion rate [/d].

The N uptake rate depends on both external and intracellular nutrients as in (Rhee 1973),

$$\text{BotAlgUpN} = \rho_{mNb} \frac{n_a + n_n}{k_{sNb} + n_a + n_n} \frac{K_{qNb}}{K_{qNb} + (q_{Nb} - q_{0Nb})} \tag{22}$$

where $\rho_{mNb}$ = the maximum uptake rate for nitrogen [mgN/m$^2$/d], $k_{sNb}$ = half-saturation constant for external nitrogen [μgN/L] and $K_{qNb}$ = half-saturation constant for intracellular nitrogen [mgN mgA$^{-1}$].

## Bottom-algae Internal Phosphorus (*IP_b*)

The change in intracellular phosphorus in bottom algal cells is calculated from

$$S_{bP} = \text{BotAlgUpP} - q_{Pb}\ \text{BotAlgDeath} - \text{BotAlgExP} \tag{23}$$

where BotAlgUpP = the maximum uptake rate of phosphorus by bottom algae (mgP/m$^2$/d), BotAlgDeath = bottom-algae death (mgA/m$^2$/d), and BotAlgExP = the bottom-algae excretion of phosphorus (mgP/m$^2$/d), which is computed as

$$\text{BotAlgExP} = k_{eb}(T)(q_{Pb} - q_{0Pb})a_b \tag{24}$$

where $k_{eb}(T)$ = the temperature-dependent bottom-algae excretion rate [/d].

The P uptake rate depends on both external and intracellular nutrients as in (Rhee 1973),

$$\text{BotAlgUpP} = \rho_{mPb} \frac{p_i}{k_{sPb} + p_i} \frac{K_{qPb}}{K_{qPb} + (q_{Pb} - q_{0Pb})} \tag{25}$$

where $\rho_{mPb}$ = the maximum uptake rate for phosphorus [mgP/m$^2$/d], $k_{sPb}$ = half-saturation constant for external phosphorus [μgP/L] and $K_{qPb}$ = half-saturation constant for intracellular phosphorus [mgP mgA$^{-1}$].

# APPENDIX A: NOMENCLATURE

| Symbol | Definition | Units |
|---|---|---|
| $[CO_3^{2-}]$ | carbonate ion | mole/L |
| $[H^+]$ | hydronium ion | mole/L |
| $[H_2CO_3^*]$ | sum of dissolved carbon dioxide and carbonic acid | mole/L |
| $[HCO_3^-]$ | bicarbonate ion | mole/L |
| $[OH^-]$ | hydroxyl ion | mole/L |
| $a_b$ | bottom algae | $mgA/m^2$ |
| *Alk* | alkalinity | eq $L^{-1}$ or $mgCaCO_3/L$ |
| $a_p$ | phytoplankton concentration | $mgA/m^3$ |
| $C_{gb}(T)$ | temperature-dependent maximum photosynthesis rate | $mgA/(m^2\ d)$ |
| $c_T$ | total inorganic carbon | mole $L^{-1}$ |
| *f* | photoperiod | fraction of day |
| $f_{dai}$ | fraction of ammonium in dissolved form in sediment layer *i* | dimensionless |
| $F_{oxna}$ | attenuation due to low oxygen | dimensionless |
| *H* | water depth | m |
| $I(0)$ | solar radiation at water surface | $cal/cm^2/d$ |
| $k(T)$ | temperature dependent first-order reaction rate | /d |
| $K_1$ | acidity constant for dissociation of carbonic acid | |
| $K_2$ | acidity constant for dissociation of bicarbonate | |
| $K_a$ | equilibrium coefficient for ammonium dissociation | |
| $k_{db}(T)$ | temperature-dependent bottom-algae death rate | /d |
| $k_e$ | light extinction coefficient | $/m^1$ |
| $k_{eb}$ | a background coefficient accounting for extinction due to water and color | /m |
| $k_{hnxb}$ | preference coefficient of bottom algae for ammonium | $mgN/m^3$ |
| $K_{Lb}$ | bottom-algae light parameter | |
| $k_{rb}(T)$ | temperature-dependent bottom-algae respiration rate | /d |
| $k_{sNb}$ | nitrogen half-saturation constant for bottom algae | μgN/L |
| $k_{sPb}$ | phosphorus half-saturation constant for bottom algae | μgP/L |
| $m_i$ | inorganic suspended solids | mgD/L |
| $m_o$ | detritus | mgD/L |
| $n_a$ | the ammonium concentration in the overlying water | $mgN/m^3$ |
| $n_{au}$ | unionized ammonia nitrogen | $mgN/m^3$ |
| $n_n$ | nitrate concentration in the overlying water | $mgN/m^3$ |
| *o* | the dissolved oxygen concentration in the overlying water | $gO_2/m^3$ |
| $P_{ab}$ | preference for ammonium as a nitrogen source for bottom algae | dimensionless |
| $PAR(z)$ | photosynthetically available radiation (PAR) at depth *z* below water surface | ly/d |
| $p_i$ | the inorganic phosphorus in the overlying water | $mgP/m^3$ |
| $p_o$ | organic phosphorus | μgP/L |
| $S_{b,i}$ | sources and sinks of constituent due to reactions for bottom algae | $mgA/m^2/d$ |

| | | |
|---|---|---|
| *t* | time | d |
| *T* | water temperature | $^{o}C$ |

# Greek:

| Symbol | Definition | Units |
|---|---|---|
| $\alpha_0$ | fraction of total inorganic carbon in carbon dioxide | dimensionless |
| $\alpha_1$ | fraction of total inorganic carbon in bicarbonate | dimensionless |
| $\alpha_2$ | fraction of total inorganic carbon in carbonate | dimensionless |
| $\alpha_i$ | effect of inorganic suspended solids on light attenuation | L/mgD/m |
| $\alpha_o$ | effect of particulate organic matter on light attenuation | L/mgD/m |
| $\alpha_p$ | linear effect of chlorophyll on light attenuation | L/μgA/m |
| $\alpha_{pn}$ | non-linear effect of chlorophyll on light attenuation | $(L/\mu gA)^{2/3}/m$ |
| $\phi_{Lb}$ | bottom-algae light attenuation | 0-1 |
| $\phi_{Nb}$ | bottom-algae nutrient attenuation factor | 0-1 |

## *APPENDIX B: CODE*

The

## *APPENDIX 1: Assessing the impact of major point sources on the validity of the homogeneous assumption*

Two simple approaches can be employed to assess whether horizontal gradients due to point source discharges could be important.

1. Direct measurements of water-quality constituents can be made at a series of locations across the stream width to establish that gradients are minimal. This can be readily done with a standard water-quality sonde. Parameters such as conductivity, temperature, pH, dissolved oxygen and turbidity are all useful for detecting bank-to-bank gradients.

2. An estimate of the mixing length, $L_m$ (m), of a point discharge to the side of a channel is provided by (Fischer et al. 1979; Rutherford 1994)

$$L_m = \frac{2}{3}\frac{UB^2}{H^{3/2}\sqrt{gS}} \qquad (26)$$

where $U$ = velocity (m/s), $B$ = width (m), $H$ = mean depth (m), $g$ = 9.81 m/s$^2$, and $S$ = channel slope (m/m).

Example: The following characteristics are generally characteristic of the Yellowstone River between Rosebud and Glendive:

$S$ = 0.0006
$B$ = 140 m
$H$ = 1.4 m
$U$ = 0.6 m/s

$$L_m = \frac{2}{3}\frac{0.6(140)^2}{(1.4)^{3/2}\sqrt{9.81(0.0006)}} = 61{,}690 \text{ m} = 61.7 \text{ km}$$